Supplementary Material for: "Orbital angular momentum based *intra*- and *inter*- particle entangled states generated via a quantum dot source"

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1 Theoretical background

In this section, we provide a theoretical insight about the different HOM interference cases reported in the Main Text. Recalling that the action of a symmetric Beam Splitter (BS) with input modes $\{a, b\}$ and output modes $\{c, d\}$ can be expressed as:

$$\begin{pmatrix} \hat{a} \\ \hat{b} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} \hat{c} \\ \hat{d} \end{pmatrix} \tag{S1}$$

Therefore, by considering two photons at the two inputs of the beam splitter, the initial state is $\hat{a}^\dagger \hat{b}^\dagger \ket{0, 0}$ and the output state results to be:

$$\frac{1}{2} \left(\hat{c}_a^\dagger \hat{c}_b^\dagger + \hat{c}_a^\dagger \hat{d}_b^\dagger - \hat{c}_b^\dagger \hat{d}_a^\dagger - \hat{d}_a^\dagger \hat{d}_b^\dagger \right) \ket{0, 0}, \tag{S2}$$

where $\ket{0, 0}$ refers to the vacuum in the two ports of the BS and the subscripts a and b to the field modes at the inputs. Two photons are indistinguishable if their state, associated to each degree of

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freedom, is the same from the point of view of the observer. In this case, when the two initial modes a and b are identical, it is not possible for the observer to discriminate which one of the two photons come out from the outputs c or d. In other words, the term $\hat{c}_a^\dagger \hat{d}_b^\dagger - \hat{c}_b^\dagger \hat{d}_a^\dagger$ vanishes.

For what concern the BS action on the polarization and Orbital Angular Momentum (OAM) states, we have that the horizontal and vertical polarizations result to be eigenstates of the BS operation since the reflection introduces a phase $\phi = \pi$ between them. On the other side, in the OAM space, the eigestates are the balanced superpositions $\frac{|m\rangle + |-m\rangle}{\sqrt{2}}$ and $\frac{|m\rangle - |-m\rangle}{\sqrt{2}}$ which are not affected by the elicity filp.

For instance, considering the case in which the waveplates change the input photons polarization to $|R\rangle$ in both of the arms, after passing through the q-plate (see Main Text, Eq. 1), at the exit of the BS we have:

$$
\begin{aligned}
\frac{1}{2}(\hat{c}_{L,-2}^\dagger - \hat{d}_{R,2}^\dagger)(\hat{c}_{R,2}^\dagger + \hat{d}_{L,-2}^\dagger)|0,0\rangle_{cd} = \\
= \frac{1}{2}(|L,-2\rangle_c |R,2\rangle_c + |L,-2\rangle_c |L,-2\rangle_d - \\
|R,2\rangle_c |R,2\rangle_d - |R,2\rangle_d |L,-2\rangle_d)
\end{aligned}
\tag{S3}
$$

In this case is not possible to extinguish the double coincidence terms and to see the HOM interference we need to enter on the q-plates with orthogonal polarization states. For example, with $|R\rangle_a$ and $|L\rangle_b$ we have:

$$
\begin{aligned}
\frac{1}{2}(\hat{c}_{L,-2}^\dagger - \hat{d}_{R,2}^\dagger)(\hat{c}_{L,-2}^\dagger + \hat{d}_{R,2}^\dagger)|0,0\rangle_{cd} = \\
= \frac{1}{\sqrt{2}}(|L,-2\rangle_c |L,-2\rangle_c - |R,2\rangle_d |R,2\rangle_d)
\end{aligned}
\tag{S4}
$$

This behaviour is experimentally observed in our setup as reported in Fig. 5 (a,b) of the Main

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Text. If instead we enter the q-plates with linearly polarized photons on both the arms, at the output of the BS we observed the complementary behavior in which we observed coincidences only when the photons have orthogonal polarization. In fact considering the input states $|H\rangle_a |H\rangle_b$ and $|H\rangle_a |V\rangle_b$, after the q-plates we have $|\Phi^+\rangle_a |\Phi^+\rangle_b$ and $|\Phi^+\rangle_a |\Phi^-\rangle_b$, then the BS output is respectively:

$$a^\dagger_{\Phi+} b^\dagger_{\Phi+} \longmapsto \frac{1}{4}(\hat{c}^\dagger_{R,2} - \hat{d}^\dagger_{L,-2} + \hat{c}^\dagger_{L,-2} - \hat{d}^\dagger_{R,2})(\hat{c}^\dagger_{L,-2} + \hat{d}^\dagger_{R,2} + \hat{c}^\dagger_{R,2} + \hat{d}^\dagger_{L,-2}) |0,0\rangle_{cd} =$$
$$= \frac{1}{4}(\sqrt{2}\,|L,-2\rangle_c |L,-2\rangle_c + \sqrt{2}\,|R,2\rangle_c |R,2\rangle_c + 2\,|L,-2\rangle_c |R,2\rangle_c + \quad \text{(S5)}$$
$$- \sqrt{2}\,|L,-2\rangle_d |L,-2\rangle_d - 2\,|L,-2\rangle_d |R,2\rangle_d - \sqrt{2}\,|R,2\rangle_d |R,2\rangle_d)$$

$$a^\dagger_{\Phi+} b^\dagger_{\Phi-} \longmapsto \frac{1}{4}(\hat{c}^\dagger_{R,2} - \hat{d}^\dagger_{L,-2} + \hat{c}^\dagger_{L,-2} - \hat{d}^\dagger_{R,2})(\hat{c}^\dagger_{L,-2} + \hat{d}^\dagger_{R,2} - \hat{c}^\dagger_{R,2} - \hat{d}^\dagger_{L,-2}) |0,0\rangle_{cd} =$$
$$= \frac{1}{4}(\sqrt{2}\,|L,-2\rangle_c |L,-2\rangle_c - \sqrt{2}\,|R,2\rangle_c |R,2\rangle_c + 2\,|L,-2\rangle_c |L,-2\rangle_d + \quad \text{(S6)}$$
$$+ \sqrt{2}\,|L,-2\rangle_d |L,-2\rangle_d + 2\,|R,2\rangle_c |R,2\rangle_d - \sqrt{2}\,|R,2\rangle_d |R,2\rangle_d)$$

According to Eq. (S5) no coincidences are expected, while in Eq. (S6) the terms in which each photon exit to different paths do not cancel out and we have a coincidences probability of $p_{c,d} = \frac{1}{2}$. The expected visibility is then $V^{th}_{\Phi+\Phi+} = 1 - 2p_{c,d} = 1$ and $V^{th}_{\Phi+\Phi-} = 0$, these are in agreement with the measured behaviours reported in Fig. 5 (c,d).

Moreover, we analyze also the case when interfering input states are neither equal nor orthogonal. Let us consider to income on the input q-plates with polarizations $|R\rangle_a$ and $|H\rangle_b$. In the first arm the generated state is $|L,-2\rangle_a$, while a VV state $|\Phi^+\rangle_b$ is generated in the second exit. Tacking into

account the BS transformation, the output beam result to be:

$$\frac{1}{2}(\hat{c}_{L,-2}^{\dagger} - \hat{d}_{R,2}^{\dagger})(\hat{c}_{R,2}^{\dagger} + \hat{d}_{L,-2}^{\dagger} + \hat{c}_{L,-2}^{\dagger} + \hat{d}_{R,2}^{\dagger}) \left| 0,0 \right\rangle_{cd} =$$

$$= \frac{1}{2\sqrt{2}} (\sqrt{2} \left| L, -2 \right\rangle_c \left| L, -2 \right\rangle_c + \left| L, -2 \right\rangle_c \left| R, 2 \right\rangle_c +$$

$$+ \left| L, -2 \right\rangle_c \left| L, -2 \right\rangle_d - \left| R, 2 \right\rangle_c \left| R, 2 \right\rangle_d - \tag{S7}$$

$$- \left| R, 2 \right\rangle_d \left| L, -2 \right\rangle_d - \sqrt{2} \left| R, 2 \right\rangle_d \left| R, 2 \right\rangle_d)$$

In this condition the probability of having both photons on the same output is $p_{c,c} + p_{d,d} = \frac{3}{4}$ while the probability of having the photons come out of different outputs is $p_{c,d} = \frac{1}{4}$. Therefore, we expect an HOM visibility of $V_{R,H}^{Th} = \frac{1}{2}$, which is consistent with the experimental one reported in the Main Text.

2 Efficiency estimation

In this section, the transmissions of all optical components are reported to estimate the efficiency of the experimental platform. The single-photon count rate $C^{te} = 4$ MHz is measured at the output of the Q-Fiber (Quandela) module with an Avalanche Photo-Diode (APD) detector with $\eta_{\det} \sim 38\%$ efficiency. Knowing the QD source is pumped with a 79 MHz-pulsed laser ($R_{\text{exe}} = 79$ MHz) and accounting for the limited efficiency of the APD, we can derive the fibered brightness of the single-photon source $\eta_{\text{fibered}} \sim 13.3\%$. The Q-Fiber module is connected to the experimental platform through optical fibers. Each connection is made with FC/PC mating sleeves, which each have a transmission of $\eta_{\text{connector}} \sim 80\%$, while the fiber-BS employed in the *inter*-particle regime has an insertion loss of $\eta_{\text{BS}} \sim 0.75$. The single-photon are then filtered in polarization with Polarizing Beam Splitters (PBS) and, since there is a residual of vertical polarized photons

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generated by the QD, the efficiency is $\eta_{\mathrm{pol}} \sim 83\%$. The major sources of losses are the q-plates which have an efficiency of $\eta_{\mathrm{q-plate}} \sim 75\%$ and the coupling of a VV beam converted to gaussian mode with a SMF $\eta_{\mathrm{coupling}} \sim 45\%$. In conclusion, the overall efficiency of the generation stage is

$$\eta_{\mathrm{gen,1}} = \eta_{\mathrm{fibered}}\eta_{\mathrm{connector}}^2\eta_{\mathrm{pol}}\eta_{\mathrm{q-plate}} = 5.30\% \text{ and } \eta_{\mathrm{gen,2}} = \eta_{\mathrm{fibered}}\eta_{\mathrm{connector}}^3\eta_{\mathrm{BS}}\eta_{\mathrm{pol}}\eta_{\mathrm{q-plate}} = 3.18\%$$

respectively for *intra-* and *inter*-particle regime, while the tomography setup results to have an efficiency of $\eta_{\mathrm{tomo}} = \eta_{\mathrm{q-plate}}\eta_{\mathrm{coupling}}\eta_{\mathrm{det}} = 12.82\%$. Using such approximated efficiency, it is possible to estimate the expected signals in the different configurations of the experiment. For the *intra*-particle entanglement, all the measurements are performed on single events, and by taking into account the 50% loss due to the second beamsplitter (BS) of the interferometer, the expected generation rate is $R_{\mathrm{intra}}^{gen} = \frac{\eta_{\mathrm{gen,1}}R_{\mathrm{exe}}}{2} = 2.09$ MHz, while the detected count rate should be $R_{\mathrm{intra}} = \eta_{tomo}R_{\mathrm{intra}}^{gen} = 267.9$ kHz. On the other side, in the *inter*-particle experiment, the measurements are performed detecting the coincidence counts, therefore the expected generation rate is $R_{\mathrm{inter}}^{gen} = \frac{\eta_{\mathrm{gen,2}}^2 R_{\mathrm{exe}}}{4 \cdot 2} = 9.98$ kHz while the detected coincidence rate is $R_{\mathrm{inter}}^{gen} = \eta_{\mathrm{tomo}}^2 R_{\mathrm{inter}}^{gen} = 164.0$ Hz, where the coefficients $\frac{1}{4}$ and $\frac{1}{2}$ are due to the first and second BS of the interferometer, respectively.

η_{fibered}	$\eta_{\mathrm{connector}}$	η_{BS}	η_{pol}	$\eta_{\mathrm{q-plate}}$	η_{coupling}	η_{det}	R_{intra}^{gen} (MHz)	R_{intra} (kHz)	R_{inter}^{gen} (kHz)	R_{inter} (Hz)
13.3%	80%	75%	83%	75%	45%	38%	2.09	267.9	9.98	164.0

Table S1 Efficiency estimation: The table showcases the experimental efficiency estimated for each loss of the apparatus. η_{fibered} represents the overall transmission efficiency of the single-photon source, including excitation laser rejection and extraction system losses. $\eta_{\mathrm{connector}}$ is the transmission efficiency of single-mode optical fiber, η_{BS} the insertion loss of the fiber-BS, while $\eta_{\mathrm{q-plate}}$ and η_{coupling} are the main losses of the implemented experimental platform due to the limited conversion efficiency of OAM modes and the higher divergence to which beams endowed with orbital angular momentum are subjected. Finally, η_{det} is the intrinsic efficiency of the avalanche photo-diode detectors employed. Using these values the expected generated and detected count rates are evaluated for both the *intra*- (R_{intra}^{gen} and R_{intra}) and *inter*-particle (R_{inter}^{gen} and R_{inter}) regime.